

Impact load identification based on local measurements

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Abstract. A new methodology for load identification is proposed. The global dynamic structural response is modeled using only pre-computed, time dependent, dynamic influence matrix, describing structural response to locally generated unit impulses. Then, the impact load identification procedure is based on distance minimization between the modeled and measured local dynamic responses in sensor locations. The theoretical background as well as numerical examples is presented.

Introduction

The objective of the paper is to continue the research carried out by [1] and present methodology and numerical examples of impact load identification on the base of local strain measurements done in few, properly chosen locations.

The problem of identification of impact localization for elastic membrane using the concept of so-called *smart layer* (thin layer with imbedded piezo-sensors) has been discussed in [2,3,4,5]. The problem discussed in this paper is formulated more generally and can be extended including physically non-linear structural behavior (e.g. plastic yielding).

Let us assume that the sensor system (e.g. piezo-transducers) distributed on the structure is able to measure and store the history of local strains development. Then, the method of corresponding load identification (the inverse problem) can be based on the so called *Virtual Distortion Method (VDM)* [6,7] making use of the *dynamic influence matrix* $D_{ij}(t)$. This matrix describes the dynamic structural response (strain development in time) in observed locations j caused by unit impulse (in time $t=0$) applied in location i . Taking into account unknown load intensities in locations i and in time instances $\tau < t$, the objective function F describing mean-square *distance* between the measured and modeled strains (in locations j and in time instances t) can be formulated. The problem of load identification leads in this case to the minimization of the F function and can be based on the gradient optimization technique, as the proposed approach allows numerically efficient, analytical sensitivity computation. The approach described above can be applied in the case of elastic structural response. In the case of elasto-plastic structural response, the analysis has to be generalized introducing additionally plastic distortions into the model (determined also on the basis of the VDM method). Similar approach has been applied to impact loads in structural adaptation [8,9].

The problem formulation (restricted to small deformations) and methodology of the solution illustrated with a numerical example is presented below.

Gradient based methodology of the impact load identification

It has been demonstrated in [10] that the VDM based description of the dynamic response of elasto-plastic truss structure making use of so-called *impulse-influence-matrix* $D_{ij}(t)$ can be applied

to load identification. The strain and stress development in element i of the structure loaded in nodes n and plastified in elements k can be expressed as follows:

$$\begin{aligned}\varepsilon_i(t) &= \sum_{\tau \leq t} \sum_n D_{in}(t-\tau) \alpha_n^0(\tau) + \sum_{\tau \leq t} \sum_k D_{ik}(t-\tau) \beta_k^0(\tau) \\ \sigma_i(t) &= E_i \left[\sum_{\tau \leq t} \sum_n D_{in}(t-\tau) \alpha_n^0(\tau) + \sum_{\tau \leq t} \sum_k (D_{ik}(t-\tau) - \delta_{ik}) \beta_k^0(\tau) \right]\end{aligned}\quad (1)$$

where E_i denotes the Young's modulus, δ_{ik} denotes Kronecker's delta, $\alpha_j^0(\tau)$ denotes load intensity (components of external load) in the node j , $\beta_k^0(\tau)$ denotes plastic distortion generated in the element k and the matrices: $D_{in}(t-\tau)$, $D_{ik}(t-\tau)$ are called the impulse (or dynamic) influence matrices, which describe: dynamical responses (strain development in time) in the element i caused by unit force applied in the node n , and by unit virtual distortions modeling plastic yield in the element k (in time instances $\tau \leq t$), respectively. The unit force in the form of the Dirac's type of impulse is used to generate structural responses which built the influence matrix. The rows of $D_{ij}(t)$ can be determined by integration of the equation of motion (e.g. through the Newmark's algorithm), calculating strains for the sequence of unit excitations (in nodes n or elements k , while these indices run through nodes potentially loaded and elements potentially plastified). The influence matrices collect all information about the structure, including the boundary conditions. Describing elasto-plastic constitutive law through the piece-wise-linear relation with hardening (shown in Fig.1) the following formula for plastic distortion development can be derived:

$$D_{ij}(0) = 0 \quad (2)$$

$$\begin{aligned}\beta_i^0(t) &= -(1-\gamma_i) \varepsilon_i^* + \sum_{\tau < t} \sum_n (1-\gamma_i) D_{in}(t-\tau) \cdot \alpha_n^0(\tau) \\ &\quad + \sum_{\tau < t} \sum_k ((1-\gamma_i) D_{ik}(t-\tau) - \delta_{ik}) \cdot \beta_k^0(\tau)\end{aligned}\quad (3)$$

where the right hand side of equation is independent of the time instance $t = \tau$. It depends only on the history of load and plastic zone development.

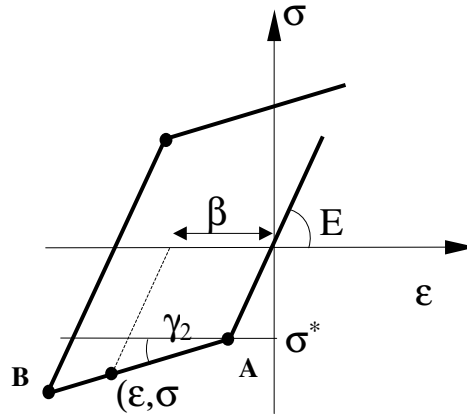


Figure. 1 Piece-wise-linear constitutive relation.

If the external load is determined ($\alpha_n^0(t)$ is given), then for each time step, after reaching the yield stress level in one structural element, we have to determine the plastic zone development (from the formula (3)) using the plastic distortions determined in this way for calculation of the right-hand sides of equation (3) in the subsequent time steps.

Now, let us assume that the external load is unknown, which means that $\alpha_n^0(t)$, describing load intensity development, together with plastic distortions, become the unknowns to be determined. Strain development measured in chosen elements is our input data allowing for solving the problem. The objective function f can be defined as the mean-square *distance* between the measured and modeled responses of strain development (in localizations m and during the period of time instances t). Then, the load identification leads to the following process of the objective function minimization (cf. [10]):

$$d_m(t) = \varepsilon_m^M(t) - \varepsilon_m(t) \quad (4)$$

$$f = \sum_t \sum_m [d_m(t)]^2 \quad (5)$$

where gradient based optimization techniques can be applied. Non-expensive sensitivity analysis is available making use of analytical formulas (1-5). Considering a fully elastic case, without plastic distortions generated due to local overloading deformations, $\varepsilon_m(t)$ are expressed (cf. (1)) as follows:

$$\varepsilon_m(t) = \sum_{\tau \leq t} \sum_n D_{mn}^P(t - \tau) \alpha_n^0(\tau) \quad (6)$$

Table.1 Heuristic optimization algorithm

Initialization: Number of all expected forces – <i>nof</i> ; number of dominant forces – <i>m</i> ; shape of the force – <i>sin</i> ; direction of the force – +, -	
Dynamic response (1)	①
Determine the objective function and its gradient $\nabla f_n(t)$	
Update force intensity $\alpha_n^0(t) = \alpha_n^0(t) + \delta f \frac{\nabla f_n(t)}{\sum_{\tau} \sum_m \sqrt{\nabla f_m(t)} \nabla f_m(t)}$	
go back to ①	
optimization steps	forces selection-every <i>i</i> iterations
	Select the set of <i>m</i> dominant forces from the set of all expected forces <i>nof</i>
	Select the dominant half-sine sections for each dominant force (cf. Fig. 2) and determine the points of corresponding extreme force values
	Apply the selected forces and go back to the next series of optimization steps.

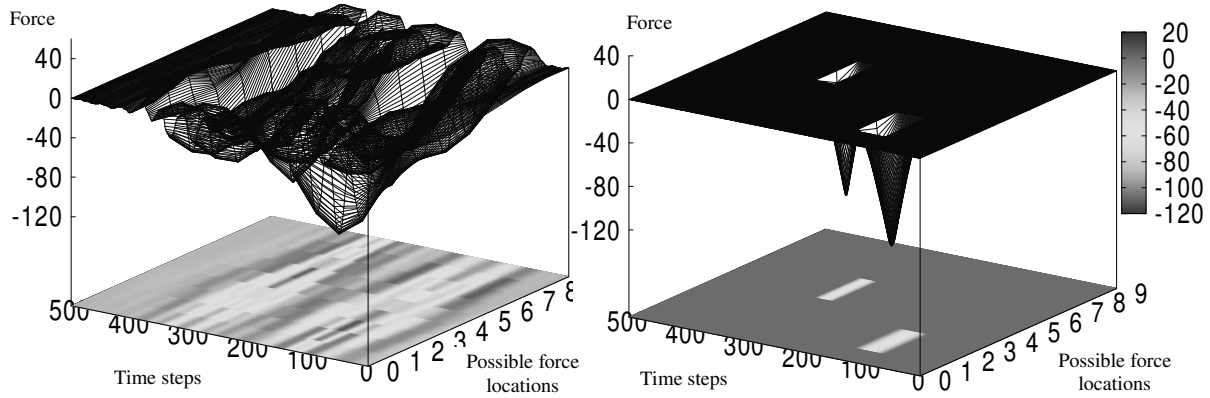


Figure. 2 Selection of forces in optimization loop.

For large structural models the above general approach is numerically expensive, but heuristic algorithms can reduce significantly the computational costs. For example, assuming that the number of impact forces, their directions (e.g. *up* or *down*) and shapes (e.g. sinus-type) are known in advance, the algorithm periodically performing (e.g. each 40 iteration steps) the selection of the most probable force locations (cf. Table 1 and Fig.2) can be applied. If only two impact forces (sinus type and acting down on the beam shown in Fig.3) are expected, the result of the selection process between 10 current estimations of developments of forces applied to 10 upper joints of the truss structure (Fig.3) is shown in Fig. 2. Two the most dominant sine-type picks are chosen.

Numerical examples

Let us illustrate the proposed methodology with the example of truss beam structure shown in Fig.2, assuming, for simplicity of demonstration that the structure is not reaching the yield stress level during the loading process. Dimensions are shown in Fig.2, while other data are specified below: material density: $7800 \left[\frac{kg}{m^3} \right]$ Young's modulus $2.1e11$ [Pa], cross-sections of elements, equal for all elements: $1e-4 \left[m^2 \right]$.

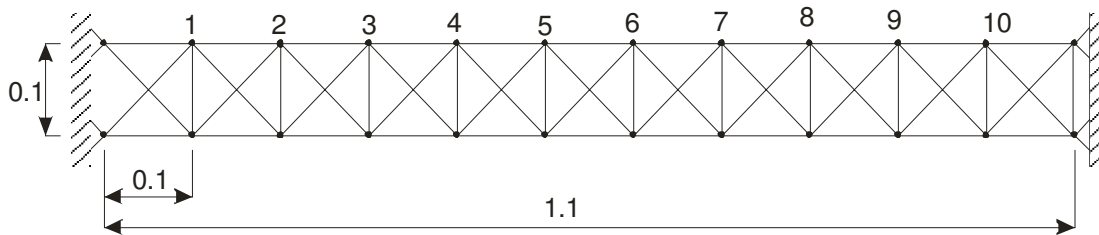


Fig. 3 Example of testing truss-beam structure

Two examples are presented, obtained via different, gradient-based optimization procedures. The first one is the steepest-descent optimization procedure with active step updating applied to the general case of impact load identification, while the second one is addressed to particular problems with predefined number of impact loads, their directions and shape-type (the heuristic algorithm, Table 1.). The unknown impact loads to be identified are modeled (in both examples) by 2 forces (cf. Fig.3), where the first one (F_3) is applied in node No. 3 and the second one (F_6) in node No. 6. The forces act in different time intervals, different frequencies and have the same amplitude.

In the first example the sensors location is shown in Fig 5. The strain developments in sensor positions (in response to impact loads shown in Figs.3, 4) are shown in Fig.6 (continuous lines). Taking these lines as the measured reference signals, the load identification procedure described above has been executed.

The obtained results are demonstrated as comparison of modeled vs. measured strains (Fig. 6) and applied vs. identified force development (Fig.7). Calculated strains are shown as the dotted lines in Fig. 6. We can see, that both lines are almost identical. The evolution of the objective function (5) is shown in Fig.8.

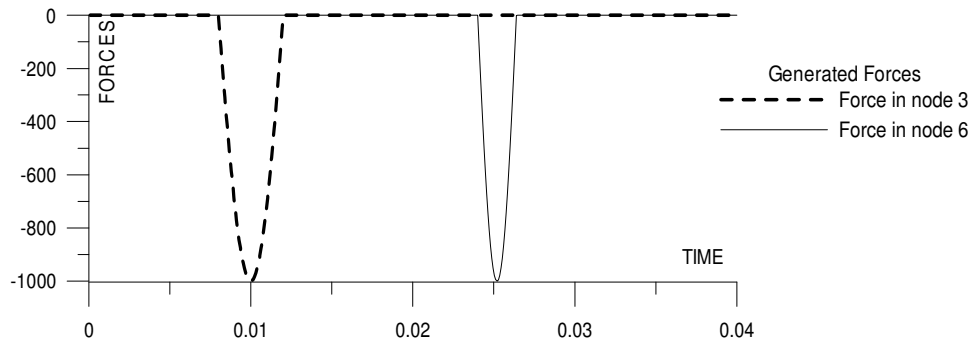


Fig. 4. History of impact loads.

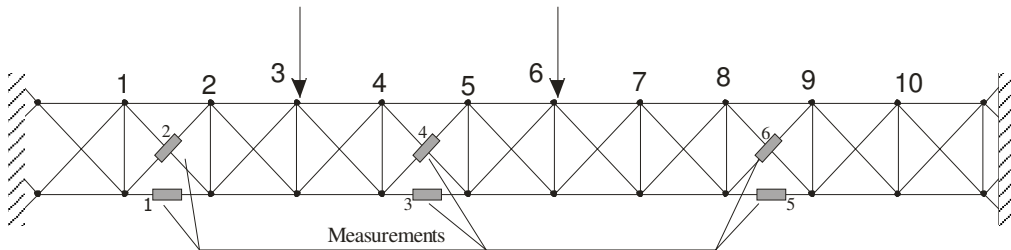


Fig. 5 Location of impact loads, and measurements first example.

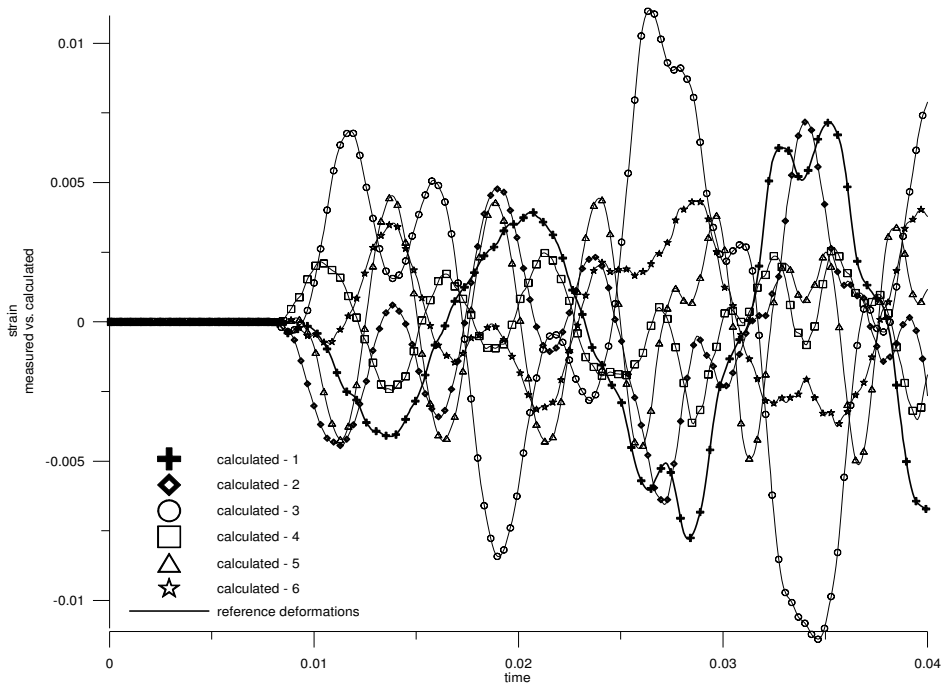


Fig. 6 „Measured” vs. modeled strains in sensor locations

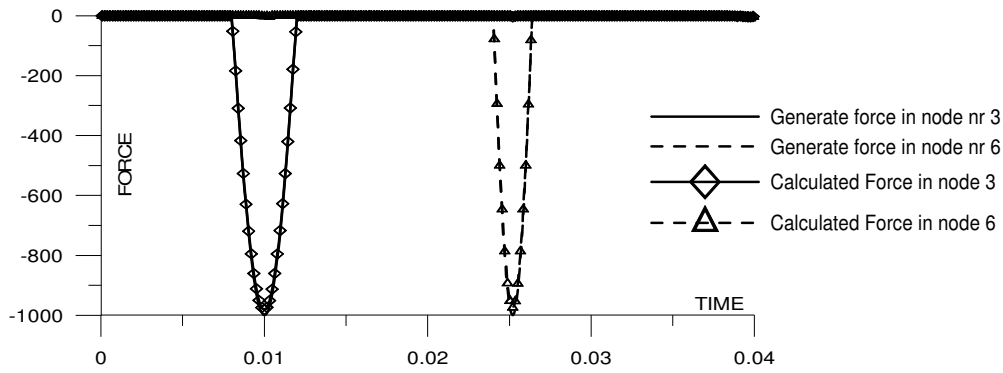


Fig. 7 Identified impact load development.

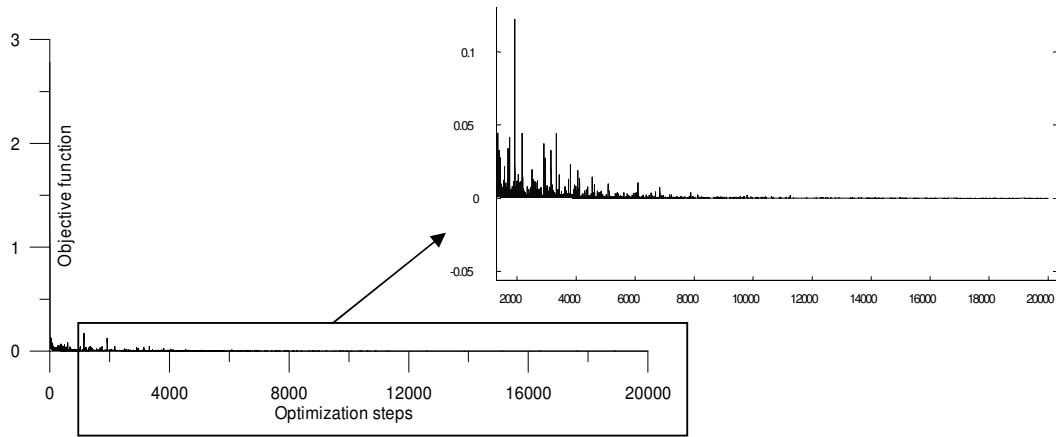


Fig. 8 The objective function evolution

In the second case (Fig.9), the numerical cost of load identification can be drastically reduced thanks to assumption that only two impact forces can be applied. Then, use of only one sensor location (e.g. in element No.3) and the modified optimization algorithm (tab. 2) is recommended.

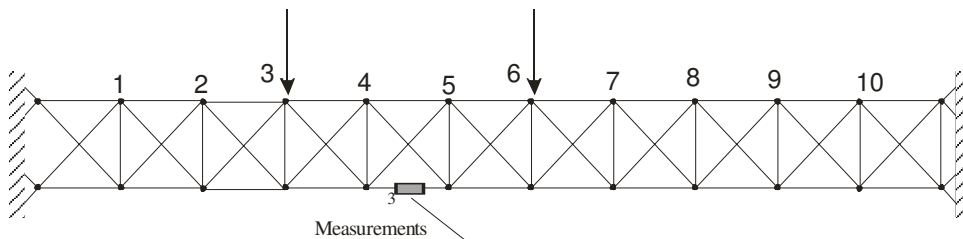


Fig. 9 Location of impact loads second example.

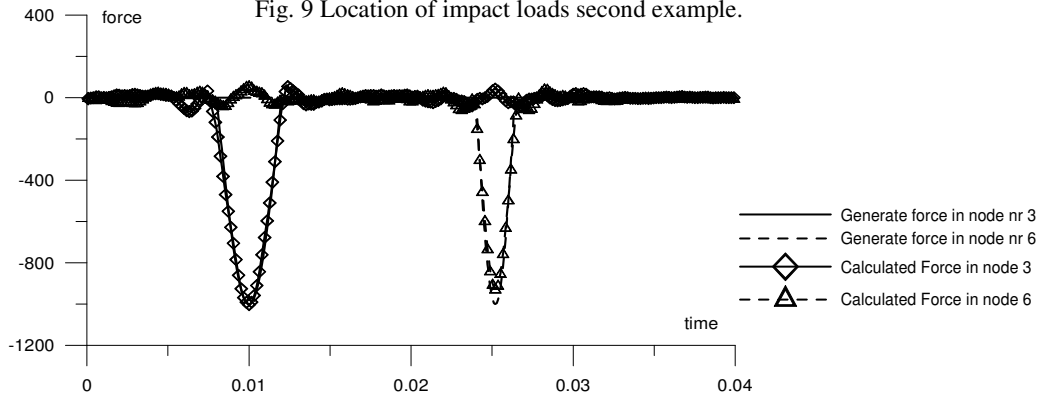


Fig. 10 Identified impact load development.

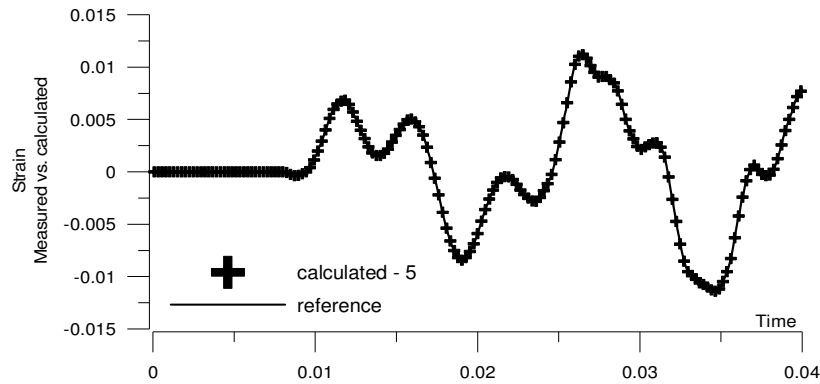


Fig. 11 „Measured” vs. modeled strains in sensor locations

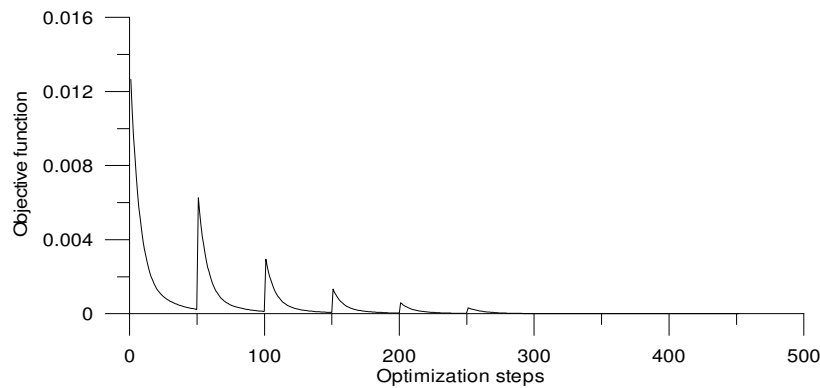


Fig. 12 The objective function evolution

The „applied” external excitation (cf. Figs 3, 9) versus identified loads is shown in Fig.10. Almost perfect match of both lines can be observed. The time reduction due to use of heuristic approach is shown in Table 2.

Table.2 Comparison of execution time

	Steepest descent algorithm	Heuristic algorithm
Number of sensors	5	1
No of optimization steps	20000	500
Computational time	2 hours	5 minutes

Application of the heuristic algorithm (and reduction of sensor number, e.g. to only one) leads to significant reduction of computation time necessary to reach satisfactory accuracy of the solution.

Conclusions

The methodology of identification of the load history in structures equipped with sensors measuring strains in few chosen elements is proposed.

In many cases, having knowledge about impact problem, it is possible to use a modified gradient-based optimization algorithms, and the number of necessary sensors observing local strain development in time can be drastically reduced (even to only one in some cases). Also, more convenient localization of sensors can be proposed in these cases.

The proposed approach makes use of gradient-based optimization technique (minimization of the objective function describing *distance* between measured and modeled signals). Analytical sensitivity analysis and automatic structural remodeling (VDM) [6,7] do not require actualization

of the global stiffness matrix and repetition of calculation of dynamic structural responses, what reduces significantly the numerical effort.

It is possible to apply the proposed concept to develop automatic systems (accident black boxes) allowing diagnosis (made *a posteriori*) concerning determination of environmental conditions causing emergency situation.

The proposed method of load identification can be also used in complex intelligent systems responding adaptively to variable environmental conditions (e.g. active adaptation to measured and identified in real time impact conditions).

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